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'Studio' mathematics for undergraduate engineers

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Abstract

Applied Mathematics for Engineering is a second year undergraduate mathematics requirement for engineering majors at Harvey Mudd College in Claremont, California since 2011. It has been jointly designed and taught by the engineering and mathematics departments. The class aims to help students develop confidence in their skill in applying mathematics to solve engineering problems and perseverance for complicated problems; to improve facility at previously learned mathematical skills and to incorporate new tools; and to develop strategic competence and better judgment on the correctness of solutions. This article describes the design principles used in creating the class and some evidence of its effectiveness.

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1 Introduction

University engineering mathematics classes typically differ from other mathematics classes in their emphasis on topics relevant to a particular engineering discipline and some use of applications from that discipline. However, often in classes (and textbooks) these application examples are straightforward mathematics problems disguised as engineering problems. At Harvey Mudd College (HMC) we found that our students were not making sufficient connections between their rigorous mathematics background and their subsequent engineering learning, and we sought to develop a class that would address this as well as increase their confidence and perseverance in drawing on mathematics in their engineering studies and careers.

Section 2 describes the unique setting in which this class was developed. Although it is not the typical environment for the education of engineers in the US or Australia, we believe that much of the content and teaching methodologies can be replicated with success in more traditional and larger programs. Section 3 describes the specific goals for the class, and Sections 4 and 5 explain its content and instructional strategies. Results and lessons learned from three years of teaching the class are described in Section 6.

2 What is Harvey Mudd College?

Founded in 1955, Harvey Mudd College in Claremont, California is a relatively young higher education institution for science, engineering and mathematics. It is very small—just 780 students—and all students are undergraduates. The priority activity for all academic staff is teaching, and experimentation with the content and delivery modes of classes is encouraged and supported.

Several things distinguish the engineering program at HMC from nearly all others in the US and Australia: it is a broad-based general engineering program without disciplinary subdivisions; all students at the college take a sizable set of common core classes; and all students pursue significant non-technical classwork for a well-rounded education. Each of these is explained in more detail below.

The HMC engineering program prepares its students for the professional world and advanced study in various disciplines through broad-based, hands-on experience in engineering analysis, synthesis and practice. Although students are able to focus upper-level elective classes and three semesters of "Engineering Clinic" (industry-sponsored capstone design) in a particular area of engineering if they wish, they do not declare specialties and must take a series of nine required intermediate-level classes in signals/systems and engineering science spanning all engineering disciplines.

All students at HMC take a common set of core classes regardless of their major field of study. In addition to three semesters of mathematics (including calculus, multi-variable calculus, linear algebra, differential equations, and probability and statistics), all take the same sequence of physics, chemistry, biology, science laboratories, computer science, systems engineering, humanities, and academic writing classes.

HMC is classified as a "liberal arts college" as all students are undergraduates and in addition to core and technical major classwork they do a significant fraction of their elective classes in the humanities, social sciences and arts. In this area of their educations, too, students have a concentration in a field of their choice as well as distribution requirements for breadth. This is fundamental to the mission of HMC:

> Harvey Mudd College seeks to educate engineers, scientists, and mathematicians, well versed in all of these areas and in the humanities and the social sciences so that they may assume leadership in their fields with a clear understanding of the impact of their work on society.

3 Class goals

In 2010, HMC revised its core curriculum to require all students to take take three semesters of mathematics instead of four. By cutting out some redundancy and a few advanced topics, the mathematics department was able to fit its core sequence in a smaller footprint. The advanced topics that were cut out were reconstituted in an optional fourth semester math class. The engineering department took advantage of the extra space afforded by this change to propose a half-semester class focused on engineering mathematics to be taken by all engineering majors in the fourth semester.

Faculty from both the engineering and mathematics departments worked together to identify areas of greatest need and to develop a set of class goals and student outcomes. Some engineering faculty were concerned that students would be getting less mathematics experience by taking only three semesters of mathematics as opposed to four, and insufficient skill at effectively applying mathematics to engineering problems was already an issue for many students. It was important that this class help students consolidate what they had previously studied and to learn how to apply that knowledge to solve engineering problems.

Consequently, we articulated the following set of class goals. As a result of taking the new class, we hope that students will:

1. develop confidence and perseverance in solving long, complex problems;
2. improve facility at previously learned mathematical skills;
3. learn several new mathematical tools (Laplace transforms, dimensional analysis, Matlab and Mathematica software);
4. develop strategic competence in selecting the appropriate mathematical tools for a particular engineering problem and better judgment on correctness of calculations.

The class, titled "Applied Mathematics for Engineering," was first taught in 2011 and has been taught annually by the authors of this article. We refer to this class by its class number, E72, in subsequent sections and describe how the above desired student learning outcomes guided the design of the class.

4 Class content

E72 is a half semester class that meets twice a week (75 minute sessions), for seven weeks. The class is designed so that each of the first six weeks focuses on one mathematics content area, along with one or two central engineering applications that make use of that mathematics (see Table 1). Homework is assigned once a week, with three to five additional substantial problems that focus on various engineering applications.

Table 1: List of emphasized topics in E72 by week

Week	Mathematics Focus	Example engineering applications used in class meetings and homework
1	Dimensional analysis and statistical data processing	Using Buckingham Pi theorem to: predict terminal velocity from experimental data; design a structure for Mars gravity; design a full-sized propeller from scale model data.
2	Linear/nonlinear systems of equations and optimization	Optimal design of a truss; thermal conduction in heat exchanger via finite difference method; electrical analogy for modeling fluid pipe networks.
3	Linear differential equations	Canonical oscillator systems: response of a filter circuit to voltage spike; response of automobile suspension; vibration modes of airplane wing.
4	More linear differential equations, Laplace Transforms	Response of skyscraper with a tuned mass damper; measurement error in a single-axis rate gyro; thermal quenching of a steel ingot.
5	Nonlinear differential equations	Stability of an automobile during a skid, of spinning bodies in space and of a DC generator circuit; simulation of a 3D inverted pendulum.
6	Optimization and differential equations	Optimal design of a tuned mass damper, of a control scheme for a heating element and of a zipline ride.

This work culminates in one final homework assignment during the seventh week that incorporates nearly all of the mathematics content from the previous six weeks. This homework assignment is written so that students must first determine a solution approach and break the problem into smaller pieces. Several analytical and computational steps are needed and multiple methods are possible for each.

Close to 75 percent of the time that we spend designing and planning this class goes to writing and selecting authentic engineering problems that lend themselves to students learning specific mathematics content. We do not hesitate to include problems in engineering fields that students have not yet encountered—we provide enough description of the application so that students understand the scenario. We also do not shield students from 'messy' mathematics (problems that involve some algebraic computation, difficult integrals, numbers that do not 'work out nicely'). As the class progresses, we expose students to increasingly realistic approximations of the kinds of the mathematical problems that they will face as professional engineers (this addresses class goals 1 and 4).

Most of the problems can be approached with methods students have learned in previous core math classes, addressing goal 2. However, after students have spent time applying familiar methods to new, complex problems, we introduce new tools, such as the Laplace transform for solving linear differential equations and Matlab and Mathematica methods for optimization, in ways that help students understand the benefits and limitations of different strategies (goals 3 and 4).

5 Instructional strategies

We chose instructional strategies for E72 that aligned with the goals of the class, current instructional practices in the engineering department, and the current literature on best practices in STEM (science, technology, engineering, mathematics) teaching and learning.

The 'studio' aspect of the class was inspired by the first engineering class, an introduction to engineering design, that all HMC engineering majors must take. In this class, all learning is project-based, and the students tackle complex, open-ended problems during class meetings while instructors are present to interact with them. Formal design methods and strategies are introduced, via small group conversations or brief mini-lectures, only once students have themselves realized the need for them in their projects [3].

Embedded within the implementation of E72 as a studio class are a number of instructional strategies that are noteworthy. Many of these instructional strategies have been around a long time and are well understood. One particularly useful source is an annotated bibliography of research on instructional strategies prepared by Froyd [2]. One strategy is *problem-based learning*, which involves introducing "relevant problems ... at the beginning of the instruction cycle ... [so as to] provide the context and motivation of the learning that follows" [4]. As mentioned in Section 4, we design our instruction so that all of the mathematical tools and knowledge are motivated by authentic engineering applications.

We try as much as possible to reduce lecturing during class to allow students to *do* mathematics and solve engineering problems. We see *active learning* as critical for building student interest and developing positive attitudes toward solving long, complex problems. When students see others struggling in class on the same challenging problems, they realize that they are not the only ones struggling and they are motivated to keep trying. When it is appropriate for the task at hand, we post helpful hints for anticipated misconceptions and areas of struggle around the classroom. When students feel stuck, they are invited to look for a hint, ask a neighbor, or ask one of us. In this way, students are also actively moving about the room and talking with each other and with us.

Because we instructors are freed from having to lecture during the entire class, we are able to circulate among students as they are working to ask probing questions, make note of what students are able to do and not able to do, and read students' body language to guess at students' feeling and attitudes toward the task at hand. This *formative assessment* allows us to adjust the pace of the class to give students enough time to productively struggle with challenging problems and avoid unproductive frustration. We also have additional tasks that we can provide to any students who finish sooner than expected. In this way, we make full use of our our low student to faculty ratio (about 25 students and two instructors in each section of E72) to ensure that every student is thoroughly engaged during class.

We design most of the active learning to be performed in *small groups* because of the great deal of evidence that collaborative, small-group learning increases student engagement and increases academic achievement [6]. We ask students to organize into groups of two, three or four when working on problems in class. Most of the time, we allow students to self-select into these groups. The furniture in the classroom is flexible enough to facilitate different arrangements of groups, and we noticed that the arrangement of furniture can lead to different kinds of interactions.

Another strategy that we use in E72 is *just-in-time learning* (not to be confused with just-in-time teaching). Just-in-time learning involves teaching students the skills and knowledge needed for a task after they have struggled with the task, rather than the other way around. For example, in the midst of the truss design task in week two, we ask students to find the optimal location of a truss joint. This task involves solving a pair of nonlinear algebraic equations. Because the class period up to that point has involved only linear systems of equations, we expect most students to try to express that system of equations as a linear algebra problem. We purposely allow them to struggle for a few moments on this task so that they realize that the equations are nonlinear before we remind them about iterative methods for solving nonlinear equations (like Newton's method) and how to use Matlab and Mathematica to perform this type of task.

Finally, we occasionally 'flip' our classroom (sometimes referred to as setting up an *inverted classroom*) through video recordings. Students are asked to watch pre-recorded, 15 minute segments of lectures before coming to class so as to free up class time for more active learning. For example we did this with the mechanics of computing Laplace transforms, an introduction to Mathematica syntax, and plotting in Matlab.

We do not claim that any of these strategies are novel. Rather, we hope to demonstrate how one can choose the appropriate instructional strategies so as to achieve a set of class goals, and to combine them into a blended practice, which we referring to here as a 'studio' class.

6 Results and effects

Rigorous assessment of the E72 class is not possible because the class was introduced along with several other changes in students' training (notably, a reduction in the number of required mathematics classes). Nevertheless, it is important to know whether the class is achieving its intended goals.

The first goal of helping students develop confidence and perseverance was surely achieved, based on students' comments on end of semester class evaluations (see Figure 1) and based on their behavior in class. Student engagement in the class was extremely high. Attendance was near perfect and students were rarely off task in class. It was frequently a challenge to get students to stop their work at the end of class so as to allow the next group of students to use the classroom. Also, we know that students were spending a large number of hours outside of class on E72. (See Figure 2).

Although we received plenty of comments on end of class evaluation forms appealing for a reduction in workload (while at the same time recognizing that it had built endurance), here is a sample of the feedback that we received in response to the question "What aspects of the teaching or content of this class do you feel were especially good?".

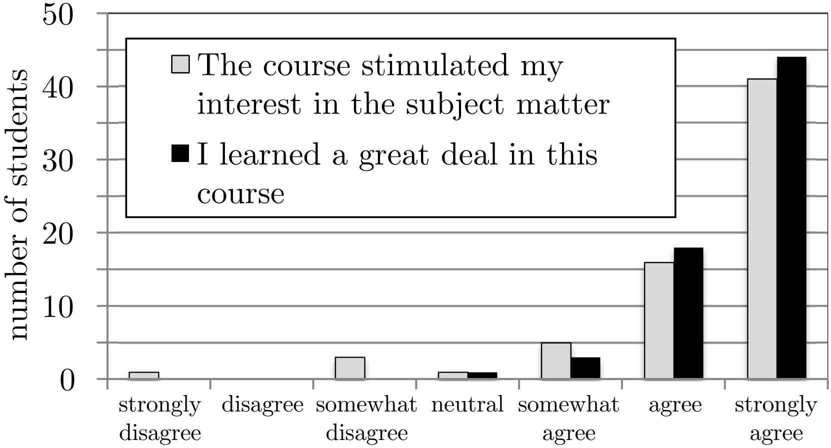

Figure 1: Students' affinity to two statements about their learning in E72, reported via end of class surveys.

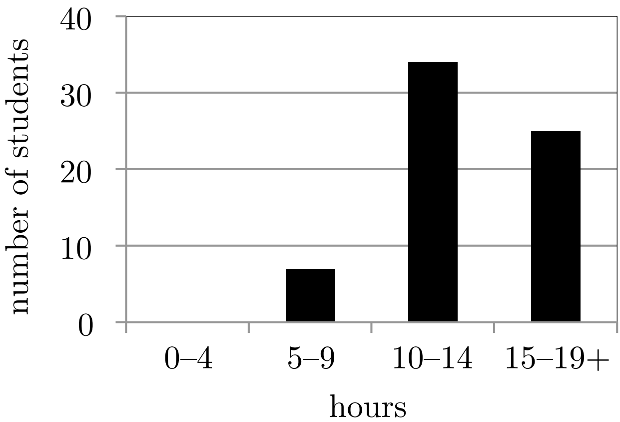

Figure 2: Number of hours spent on E72 outside of class per week, as reported by students on end of class surveys (6–9 hours is expected given the class credit hours).

> "I absolutely loved how everything we did in class had some practical application to the real world and how these problems incorporated almost everything we've learned in our math classes."

> "The in class projects were fun and offered a lot of insight into ways to approach problems."

> "I really enjoyed having two professors that gave different perspectives on the material. Having two professors was also helpful in answering everyone's questions in class."

> "I thought working on problems in class was very helpful because I could get immediate feedback/help when I ran into an issue or didn't understand something. Homework was difficult, but very effective at teaching concepts and rewarding in the end."

Recently, we administered a survey to graduating engineering majors, asking them to reflect back on what they had learned in E72 and how it affected their studies at HMC. Here are some of the comments that were submitted:

> "[Through E72] I became comfortable using programs to help solve math problems. I also became comfortable with solving lengthy problems. I learned how to break down large problems into smaller, approachable steps."

> "[E72] really got me interested in Matlab, which I built upon and ended up doing all Matlab coding for 2 clinic projects and working at SpaceX. It really helped develop my endurance for long problems."

Feedback from engineering faculty who work with students subsequent to them taking E72 also confirmed some success:

> "E72 definitely helps, especially for the stronger students. For the weaker ones, it also helps in that at least they remember seeing the material, even if they don't recall the exact details."

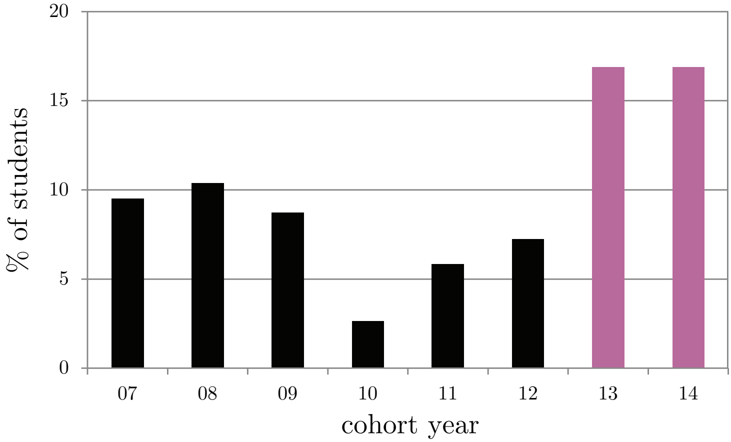

Figure 3: Percent of engineering majors choosing a mathematics-focused technical elective in each year after taking E72. The cohorts graduating in 2013 and 2014 were the first to take E72 (the 2015 cohort is only now starting technical elective classes).

> "Perhaps the strongest indicator that E72 is having a positive impact is the general sense among students that modeling is mathematics, and that modeling can be used as tool for understanding how systems work, or at least, for approximating and predicting behavior."

We also have some evidence that students have been taking more advanced mathematics classes as a result of having taken E72 (see Figure 3).

7 Key lessons

One of the main reasons E72 has been a success is that it was designed from the outset by both engineering and mathematics academic staff. Combining both engineering and mathematics expertise allowed us to ensure that the problems and examples used in class represented authentic engineering practices *and* rigorous mathematical practices. Though the investment of time was high, especially for the curation and creation of these authentic and rigorous problems, the results are very positive. Once class goals (student learning outcomes) were articulated, then the class content and method of delivery was chosen to best meet those goals. Consequently, we were able to blend a variety of instructional practices to create a 'studio' class experience that led to demonstrably positive student outcomes.

Many of the class goals will be similar to those at other institutions, so that even though this class was designed to fit the specific needs of engineering majors at Harvey Mudd College, we believe many of the same ideas and principles can be applied to other contexts. The findings of the SCALE-UP Project showed that this kind of collaborative studio-style instruction can be used for large class sizes [1]. Specific recommendations for how to help different departments to work together to design interdisciplinary learning is discussed in a summary report by Project Kaleidoscope [5].

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